

**Attendance Problems.**

1. Draw  $\overline{AB}$  and  $\overline{AC}$ , where  $A$ ,  $B$ , and  $C$  are noncollinear.
  
2. Draw opposite rays  $\overrightarrow{DE}$  and  $\overrightarrow{DF}$ .

**Solve each equation.**

3.  $2x + 3 + x - 4 + 3x - 5 = 180$

4.  $5x + 2 = 8x - 10$

Vocabulary		
angle	right angle	vertex
interior of an angle	straight angle	exterior of an angle
congruent angles	measure	angle bisector
degree	acute angle	obtuse angle

- I can name and classify angles.
- I can measure and construct angles and angle bisectors.

**Common Core**

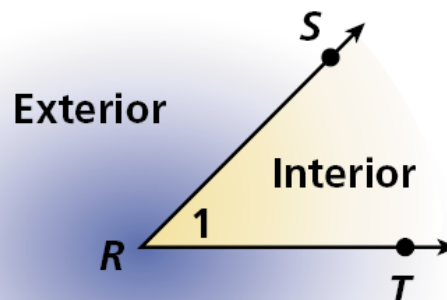
- **CC.9-12.G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometry software, etc.).
- **CC.9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

A transit is a tool for measuring angles. It consists of a telescope that swivels horizontally and vertically. Using a transit, a surveyor can measure the *angle* formed by his or her location and two distant points.

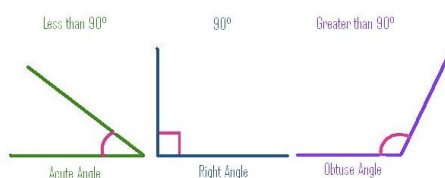
An **angle** is a figure formed by two rays, or sides, with a common endpoint called the **vertex** (plural: *vertices*). You can name an angle several ways: by its vertex, by a point on each ray and the vertex, or by a number.

The set of all points between the sides of the angle is the **interior of an angle**. The **exterior of an angle** is the set of all points outside the angle.

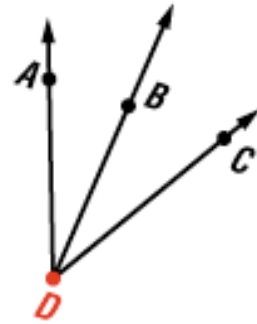
**Angle Name**  
 $\angle R$ ,  $\angle SRT$ ,  $\angle TRS$ , or  $\angle 1$



You cannot name an angle just by its vertex if the point is the vertex of more than one angle. In this case, you must use all three points to name the angle, and the middle point is always the vertex.



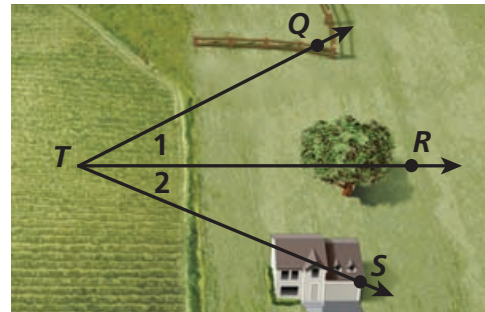
**Video Example 1:** Name three angles in the diagram.



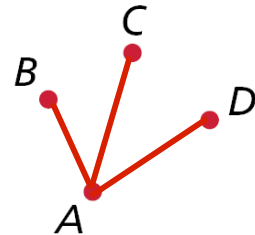
### 1 Naming Angles

A surveyor recorded the angles formed by a transit (point  $T$ ) and three distant points,  $Q$ ,  $R$ , and  $S$ . Name three of the angles.

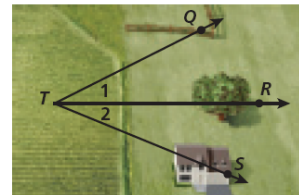
$\angle QTR$ ,  $\angle QTS$ , and  $\angle RTS$



**Example 1:** A surveyor recorded the angles formed by a transit (point  $A$ ) and three distant points,  $B$ ,  $C$ , and  $D$ . Name three of the angles.



**5. Guided Practice.** Write the different ways you can name the angles in the diagram.



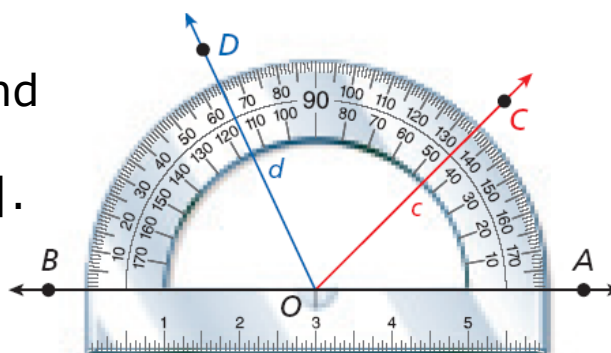
**6.** How do you measure an angle?

### Postulate 1-3-1 Protractor Postulate

Given  $\overleftrightarrow{AB}$  and a point  $O$  on  $\overleftrightarrow{AB}$ , all rays that can be drawn from  $O$  can be put into a one-to-one correspondence with the real numbers from 0 to 180.

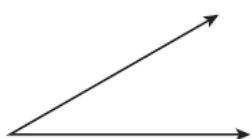
You can use the Protractor Postulate to help you classify angles by their measure. The measure of an angle is the absolute value of the difference of the real numbers that the rays correspond with on a protractor.

If  $\overrightarrow{OC}$  corresponds with  $c$  and  $\overrightarrow{OD}$  corresponds with  $d$ ,  
 $m\angle DOC = |d - c|$  or  $|c - d|$ .



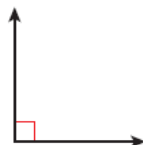
### Types of Angles

#### Acute Angle



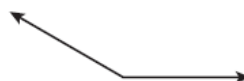
Measures greater than  $0^\circ$  and less than  $90^\circ$

#### Right Angle



Measures  $90^\circ$

#### Obtuse Angle



Measures greater than  $90^\circ$  and less than  $180^\circ$

#### Straight Angle

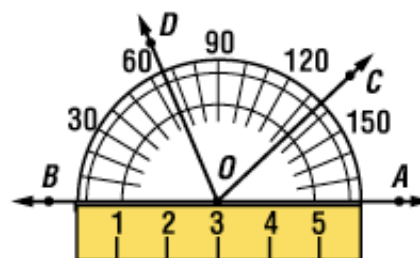


Formed by two opposite rays and measures  $180^\circ$

**Video Example 2:** Find the measure of each angle. Then classify each as acute, right, or obtuse.

A.  $m\angle BOD$

B.  $m\angle COD$

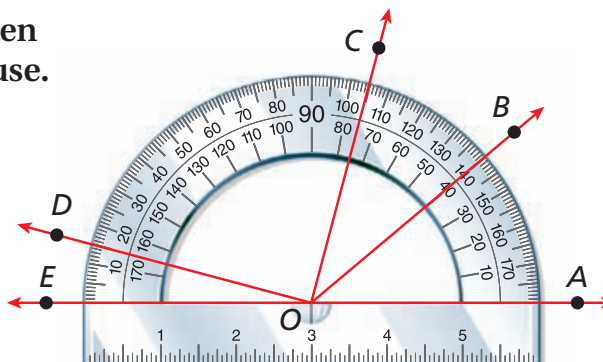


## 2 Measuring and Classifying Angles

Find the measure of each angle. Then classify each as acute, right, or obtuse.

**A**  $\angle AOD$   
 $m\angle AOD = 165^\circ$   
 $\angle AOD$  is obtuse.

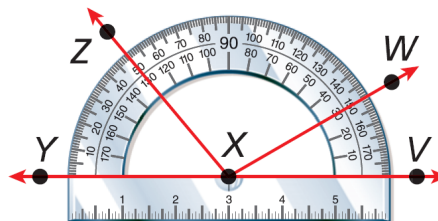
**B**  $\angle COD$   
 $m\angle COD = |165 - 75| = 90^\circ$   
 $\angle COD$  is a right angle.



**Example 2:** Find the measure of each angle. Then classify each as acute, right, or obtuse.

A.  $m\angle WXV$

B.  $m\angle ZXW$

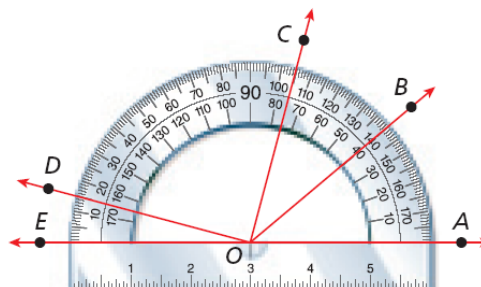


**Guided Practice.** Find the measure of each angle. Then classify each as acute, right, or obtuse.

7.  $m\angle BOA$

8.  $m\angle DOB$

9.  $m\angle EOC$



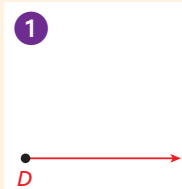
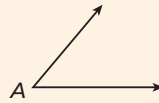
10. What are congruent angles?

11. In a diagram, how do you show two angles are congruent?

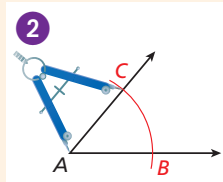


### Construction Congruent Angle

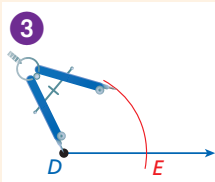
Construct an angle congruent to  $\angle A$ .



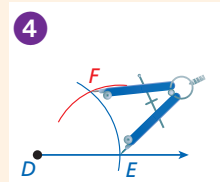
Use a straightedge to draw a ray with endpoint  $D$ .



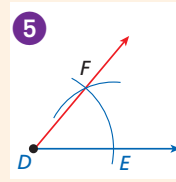
Place the compass point at  $A$  and draw an arc that intersects both sides of  $\angle A$ . Label the intersection points  $B$  and  $C$ .



Using the same compass setting, place the compass point at  $D$  and draw an arc that intersects the ray. Label the intersection  $E$ .

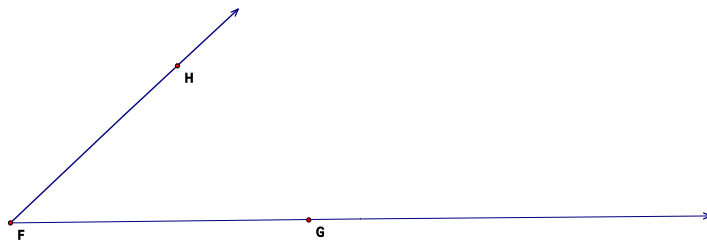


Place the compass point at  $B$  and open it to the distance  $BC$ . Place the point of the compass at  $E$  and draw an arc. Label its intersection with the first arc  $F$ .



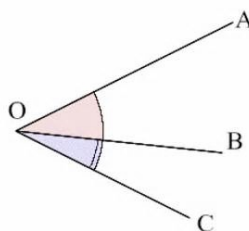
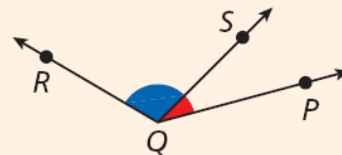
Use a straightedge to draw  $\overrightarrow{DF}$ .

$$\angle D \cong \angle A$$

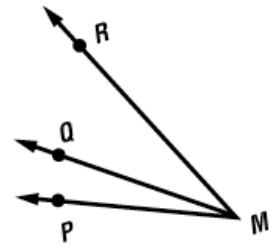


### Postulate 1-3-2 Angle Addition Postulate

If  $S$  is in the interior of  $\angle PQR$ , then  
 $m\angle PQS + m\angle SQR = m\angle PQR$ .  
 ( $\angle$  Add. Post.)



**Video Example 3.**  $m\angle PMR = 65^\circ$  and  $m\angle QMR = 38^\circ$ . Find  $m\angle PMQ$ .



### 3 Using the Angle Addition Postulate

$m\angle ABD = 37^\circ$  and  $m\angle ABC = 84^\circ$ . Find  $m\angle DBC$ .

$$m\angle ABC = m\angle ABD + m\angle DBC$$

$$84^\circ = 37^\circ + m\angle DBC$$

$$\underline{-37} \quad \underline{-37}$$

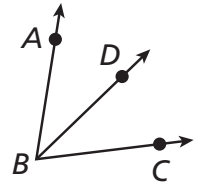
$$47^\circ = m\angle DBC$$

*∠ Add. Post.*

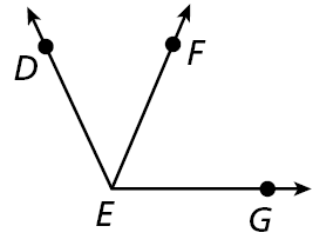
*Substitute the given values.*

*Subtract 37 from both sides.*

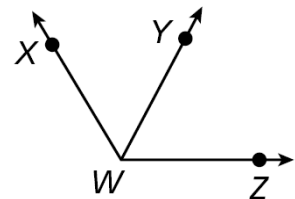
*Simplify.*



**Example 3.**  $m\angle DEG = 115^\circ$ , and  $m\angle DEF = 48^\circ$ . Find  $m\angle FEG$ .



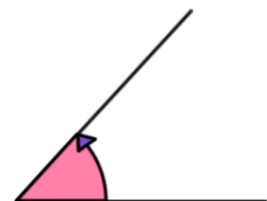
12. **Guided Practice:**  $m\angle XWZ = 121^\circ$  and  $m\angle XWY = 59^\circ$ . Find  $m\angle YWZ$ .



(p 25) 11, 12, 13, 15-

13. What is an angle bisector?

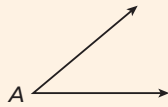
Sketchpad: Constructing an angle bisector.



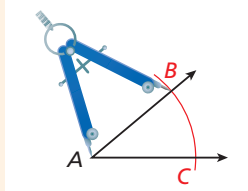


### Construction Angle Bisector

Construct the bisector of  $\angle A$ .

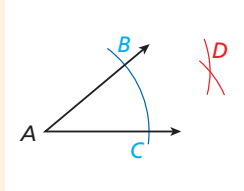


1



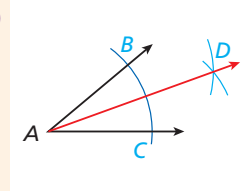
Place the point of the compass at  $A$  and draw an arc. Label its points of intersection with  $\angle A$  as  $B$  and  $C$ .

2



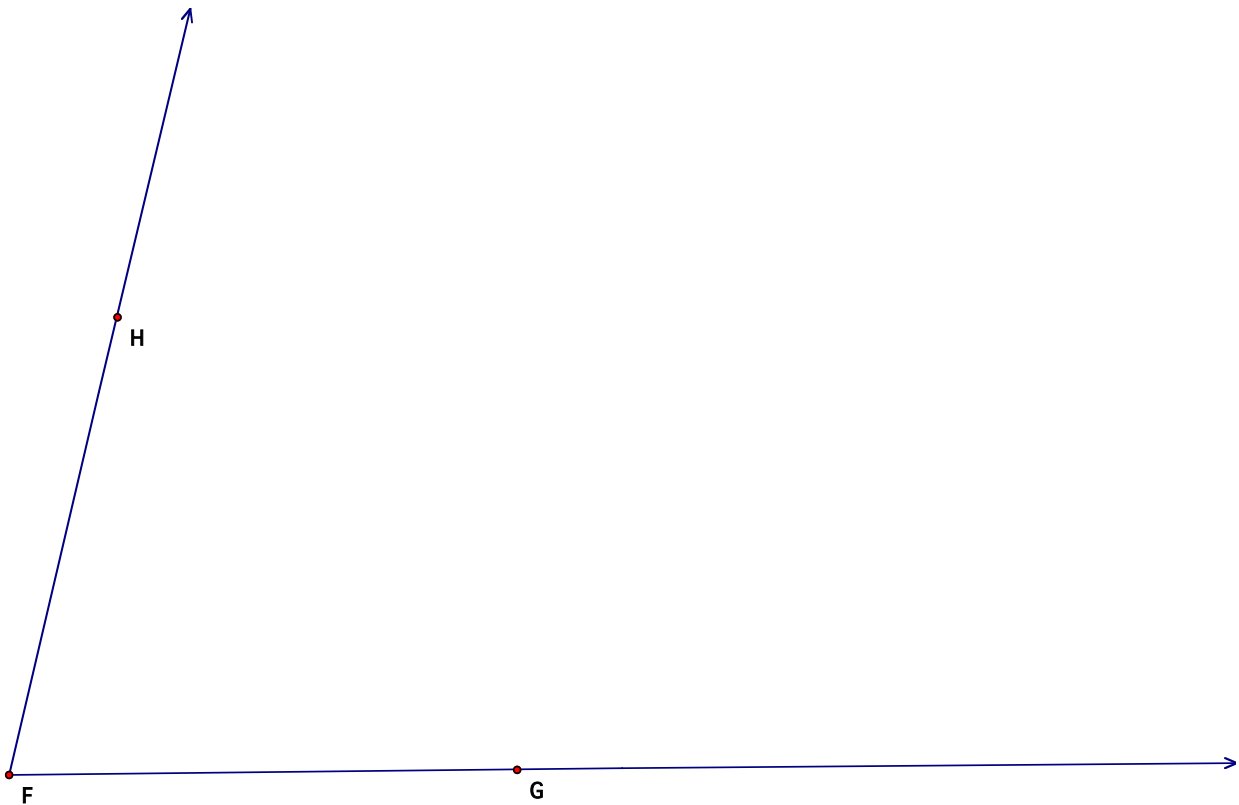
Without changing the compass setting, draw intersecting arcs from  $B$  and  $C$ . Label the intersection of the arcs as  $D$ .

3



Use a straightedge to draw  $\overrightarrow{AD}$ .

$\overrightarrow{AD}$  bisects  $\angle A$ .



**Q:** What do you call people who are in favor of tractors?

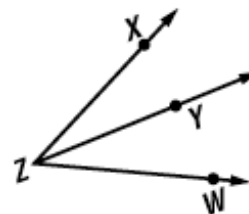
**A:** Protractors!

There are bugs and then there are bugs. And then there are bugs. -- *Karl Lehenbauer*



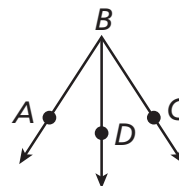
**Video Example 4:**  $\overline{ZY}$  bisects

$\angle XZY = (7x + 1)^\circ$  &  $m\angle YZW = (5x + 11)^\circ$ . Find  $m\angle XZW$ .



**4 Finding the Measure of an Angle**

$\overrightarrow{BD}$  bisects  $\angle ABC$ ,  $m\angle ABD = (6x + 3)^\circ$ , and  $m\angle DBC = (8x - 7)^\circ$ . Find  $m\angle ABD$ .



**Step 1** Find  $x$ .

$$\begin{aligned} m\angle ABD &= m\angle DBC \\ (6x + 3)^\circ &= (8x - 7)^\circ \\ \underline{+ 7} \quad \underline{+ 7} & \\ 6x + 10 &= 8x \\ \underline{- 6x} \quad \underline{- 6x} & \\ 10 &= 2x \\ \frac{10}{2} &= \frac{2x}{2} \\ 5 &= x \end{aligned}$$

*Def. of  $\angle$  bisector*

*Substitute the given values.*

*Add 7 to both sides.*

*Simplify.*

*Subtract  $6x$  from both sides.*

*Simplify.*

*Divide both sides by 2.*

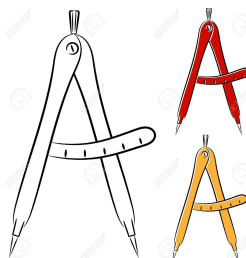
*Simplify.*

**Step 2** Find  $m\angle ABD$ .

$$\begin{aligned} m\angle ABD &= 6x + 3 \\ &= 6(5) + 3 \\ &= 33^\circ \end{aligned}$$

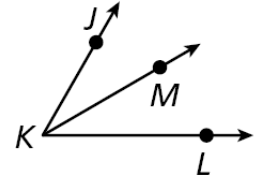
*Substitute 5 for  $x$ .*

*Simplify.*



**Example 4:**  $\overline{KM}$  bisects

$\angle JKL$ ,  $m\angle JKM = (4x + 6)^\circ$ , &  $m\angle (7x - 12)^\circ$ . Find  $m\angle JKM$ .



**Guided Practice.** Find the measure of each angle.

14.  $\overline{QS}$  bisects  $\angle PQR$ ,  $m\angle PQS = (5y - 1)^\circ$ , &  $m\angle PQR = (8y + 12)^\circ$ . Find  $m\angle PQS$ .

15.  $\overline{JK}$  bisects  $\angle LJM$ ,  $m\angle LJK = (-10x + 3)^\circ$  &  $m\angle KJM = (-x + 21)^\circ$ . Find  $m\angle LJM$ .

**1-3 Measuring and constructing angles:** (p 25) 11, 12, 13, 15-18, 20, 30, 32, 33, 37, 39.